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**A new axiomatic approach
to the evaluation of population health**

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Abstract

We explore in this paper the implications of ethical and operational principles for the evaluation of population health. We formalize those principles as axioms for social preferences over distributions of health for a given population. We single out several focal population health evaluation functions, which represent social preferences, as a result of combinations of those axioms. Our results provide rationale for popular theories in health economics (such as the unweighted aggregation of QALYs or HYE, and generalizations of the two, aimed to capture concerns for distributive justice) without resorting to controversial assumptions over individual preferences.

Keywords: population health, QALYs, HYE, axioms.

JEL Classification: D63, I10

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1 Introduction

The goal of all health services activities and programs is to improve the health of people. Such a goal places a central role to the definition and measurement of benefit in health care, as well as how it should be distributed in a population. It is not surprising that, over the years, there has been considerable interest and activity in developing methods to measure quantitatively the health status of individuals and populations (e.g., Torrance, 1986). Economists, practitioners and social scientists alike have long been concerned with this issue. Attempts to develop appealing measures to evaluate the health of a population abound in the literature (e.g., Plinskin et al., 1980; Torrance, 1986; Mehrez and Gafni, 1989; Wagstaff, 1991; Bleichrodt, 1995, 1997; Williams, 1997; Dolan, 1998, 2000; Murray et al., 2002; Bleichrodt et al., 2004; Guerrero and Herrero, 2005; Østerdal, 2005; Fleurbaey and Schokkaert, 2009).

The purpose of this paper is to take the axiomatic approach to the evaluation of population health, a somewhat unexplored approach in the health economics literature, in contrast to many other fields in economics. An axiomatic study begins with the specification of a domain of problems, and the formulation of a list of desirable properties (axioms) of solutions for the domain, whereas it ends with (as complete as possible) descriptions of the families of solutions satisfying various combinations of the properties (e.g., Thomson, 2001). An axiomatic study often results in characterization theorems. They are theorems identifying a particular solution, or perhaps a family of solutions, as the only solution or family of solutions, satisfying a given list of axioms. This is precisely what we aim to do in this paper. In other words, we aim to list some appealing axioms for the evaluation of population health and then derive precise measures to evaluate the health of a population.

It has been frequently argued that the benefit that a patient derives from a particular health care intervention is defined according to two dimensions: quality of life and quantity of life. We endorse such assumption in our model. More precisely, we assume that the distribution of health in a population is defined by a collection of duplets, each indicating the status that an agent of the population achieves in the two dimensions: health (quality of life) and time (quantity of life). We shall refer to those distributions as *population health distributions*. We, however, departure from

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earlier contributions within the health economics literature, to deal with the evaluation of health distributions, which presume an implicit relationship between the two dimensions at the individual level (e.g., Bleichrodt, 1995, 1997; Østerdal, 2005; Bleichrodt et al., 2004; Doctor et al., 2009). In other words, we do not assume from the outset the existence of an index summarizing the information of each duplet. Nevertheless, as we shall see later, we implicitly derive indices of that sort as a result of combining our axioms.

Our ultimate goal in this paper will be to single out, as a result of combining several axioms reflecting ethical and operational principles, specific measures to evaluate population health distributions. The key tool of our work to achieve that goal will be what we call a *population health evaluation function* (“PHEF” hereafter). A PHEF is a mapping that associates with each population health distribution a real number intended to perform comparisons among them and, hence, reflect social preferences over those distributions. We shall single out several PHEFs, each reflecting well-known natural views in the economic appraisal of health care programmes for resource allocation decisions. The axiomatic characterizations leading to these PHEFs will allow us to scrutinize their relative virtues by means of comparing the principles (axioms) that drive each of them.

One of the PHEFs we shall single out is the so-called *linear QALY* function, which evaluates population health by means of the unweighted aggregation of quality-adjusted life-years (QALYs). In contrast to the literature dealing with this focal and common procedure to aggregate health benefits in economic evaluations of health care, we derive it without imposing any structure on individual preferences over health. As a result, we avoid one of the main criticisms of the QALY measure, namely that it relies on restrictive assumptions over individual preferences (e.g., Loomes and McKenzie, 1989). We shall also single out the so-called *linear HYE* function, which evaluates population health by means of the unweighted aggregation of healthy years equivalents (HYEs). In doing so, we shall be able to scrutinize further the similarities and differences between these two focal concepts (HYEs and QALYs) in health economics, an aspect that has received considerable attention in the literature (e.g., Mehrez and Gafni, 1989; Culyer and Wagstaff, 1993; Gafni et al., 1993; Bleichrodt, 1995).

Both PHEFs (linear QALY and linear HYE) dismiss a concern for distributive justice in the evaluation process of population health distributions. They simply focus on the aggregate number (of QALYs or HYE) that a distribution delivers. We also characterize in this paper PHEFs modifying the previous two in order to capture concerns for distributive justice. As we shall

see later, the resulting PHEFs, which are also inspired by classical contributions in the economics literature, are closely connected to other proposals in the literature on health economics.

The rest of the paper is organized as follows. In Section 2, we introduce the model and the basic axioms we consider. In Section 3, we introduce and characterize two focal PHEFs. We generalize them, and characterize the ensuing PHEFs, in Section 4. We discuss the results and some possible extensions in Section 5. For a smooth passage, we defer the proofs and provide them in an appendix.

2 The preliminaries

We consider a policy maker who has to make a choice between distributions of health for a population of fixed size $n \geq 3$. We identify the population (society) with the set $N = \{1, \dots, n\}$. The health of each individual in the population will be described by a duplet indicating the level achieved in two parameters: quality of life and quantity of life. Assume that there exists a set of possible health states, A , defined generally enough to encompass all possible health states for everybody in the population. We emphasize that A is an abstract set without any particular mathematical structure.¹ Quantity of life will simply be described by a set of nonnegative real numbers, $T \subset \mathbb{R}$. In what follows, and unless otherwise stated, we assume that $T = [0, +\infty)$. Formally, let $h_i = (a_i, t_i) \in A \times T$ denote the health duplet of individual i .² A population health distribution (or, simply, a health profile) $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)]$ specifies the health duplet of each individual in society. We denote the set of all possible health profiles by H , i.e., H is the n -Cartesian product of the set $A \times T$. Even though we do not impose a specific mathematical structure on the set A , we assume that it contains a specific element, a_* , which we refer to as *perfect health* and which is univocally identified, as a “superior” state, by all agents in the population.

The policy maker’s preferences (or social preferences) over health profiles are expressed by a preference relation \succsim , to be read as “at least as preferred as”. As usual, \succ denotes strict preference and \sim denotes indifference. We assume that the relation \succsim is a weak order, i.e., it is complete (for each

¹A could for instance refer to the resulting multidimensional health states after combining the levels of each dimension of a categorical measure, such as EQ-5D, in all possible ways.

²For ease of exposition, we establish the notational convention that $h_S \equiv (h_i)_{i \in S}$, for each $S \subset N$.

health profiles h, h' , either $h \succsim h'$, or $h' \succsim h$, or both) and transitive (if $h \succsim h'$ and $h' \succsim h''$ then $h \succsim h''$).

A *population health evaluation function* (PHEF) is a real-valued function $P : H \rightarrow \mathbb{R}$. We say that P represents \succsim if

$$P(h) \geq P(h') \Leftrightarrow h \succsim h',$$

for each $h, h' \in H$. Note that if P represents \succsim then any (continuous) strictly increasing transformation of P would also do so.

The model we just outlined has several possible interpretations:

For instance, the pair (a_i, t_i) could be identifying an agent having a chronic (or “average”) health state a_i throughout a (remaining) lifetime of length t_i . In this sense, the model could be used to express a social planner’s preference over societies (of the same size) with different distributions of the citizens health states and (remaining) life years. Alternatively, we could interpret a_i as a vector itself $a_i = (a_i^1, \dots, a_i^{t_i})$ indicating the health state of agent i at each period of her remaining lifetime t_i .³

We may also think of the scenario in which the planner launches an intervention and this intervention results in a health state a_i for a period of time t_i for each agent i in the society, relative to some status-quo distribution. Different interventions can then be compared on the basis of their resulting distributions of health states and time periods.⁴

Finally, we could assume that a_i reflects both the quantity and quality of i ’s health state, whereas t_i reflects the income of agent i . In doing so, the social planner’s preference would not just be defined over health distributions, but rather over a proxy of welfare distributions, provided one interprets welfare as a combination of health and income.⁵

2.1 Basic structural axioms

We now list the axioms for social preferences that we consider in this paper. As we shall see, each of them will reflect an ethical or an operational principle.

³Note that no assumption from the model precludes us from interpreting A as a multidimensional space.

⁴In these first two interpretations, some of the axioms we shall consider later would be less controversial if the population is assumed to reflect a cohort of agents with the same age.

⁵This interpretation is reminiscent of the approach taken by Fleurbaey and Shokkaert (2009).

Our first axiom, *anonymity*, is a standard formalization of the principle of *impartiality*, which refers to the fact that ethically irrelevant information is excluded from the evaluation process. In other words, the identity of agents should not matter and the evaluation of the population health should depend only on the list of quality-quantity duplets, not on who holds them. Formally, let Π^N denote the class of bijections from N into itself. Then,

ANON: $h \sim h_\pi$ for each $h \in H$, and each $\pi \in \Pi^N$.

The next axiom, *separability*, underlies the use of incremental analysis in cost-effectiveness analysis, which implies that individuals for whom two treatments yield the same health should not influence the relative evaluation of these treatments (e.g., Gold et al., 1996; Turpcu et al., 2011). More precisely, it says that if the distribution of health in a population changes only for a subgroup of agents in the population, the relative evaluation of the two distributions should only depend on that subgroup.⁶ Formally,

SEP: $[h_S, h_{N \setminus S}] \succsim [h'_S, h_{N \setminus S}] \Leftrightarrow [h_S, h'_{N \setminus S}] \succsim [h'_S, h'_{N \setminus S}]$, for each $S \subseteq N$, and $h, h' \in H$.

Continuity is the adaptation to our context of a usual axiom. It says that for fixed distributions of health states, the population health ordering is smooth in lifetimes. In spite of being an apparently technical condition, it is also an ethically attractive axiom. It models *non-arbitrariness* of the social preferences.

CONT: Let $h, h' \in H$, and $h^{(k)}$ be a sequence in H such that, for each $i \in N$, $h_i^{(k)} = (a_i, t_i^{(k)}) \rightarrow (a_i, t_i) = h_i$. If $h^{(k)} \succsim h'$ for each k , then $h \succsim h'$.

The next axiom, *perfect health superiority*, refers to the superiority of the state of perfect health. More precisely, it says that replacing the health status of an agent by that of perfect health, ceteris paribus, cannot worsen the evaluation of the population health. It constitutes the closest approximation in our context to the Pareto principle of optimality, and it also reflects a certain notion of *solidarity*, a principle with a long tradition of use in the theory of justice (e.g., Moreno-Ternero and Roemer, 2006).

PHS: For each $h = [h_1, \dots, h_n] \in H$ and $i \in N$, let $h_i^* = (a_*, t_i)$. Then, $[h_i^*, h_{N \setminus \{i\}}] \succsim h$.

⁶The notion of separability has a long tradition of use in models of cooperative decision making (e.g., Moulin, 1988).

A somewhat related axiom comes next. *Time monotonicity at perfect health* says that if each agent is at perfect health, increasing the time dimension is strictly better for society. Formally,

TMPH: If, for each $i \in N$, $t_i \geq t'_i$, at least one strict, then $[(a_*, t_1), \dots, (a_*, t_n)] \succ [(a_*, t'_1), \dots, (a_*, t'_n)]$.

The previous two axioms convey some sort of desirability of the perfect health state. The next axiom, however, puts a cap on it. More precisely, *non-desirable death* states that society (strictly) improves by replacing an agent with perfect health and zero lifetime, with any other agent having a positive lifetime (independently of her health state). Formally,

NDD: For each $h \in H$ and $i \in N$ such that $t_i > 0$, $h \succ [h_{N \setminus \{i\}}, (a_*, 0)]$.

The next axiom, *social zero condition*, also refers to the case of zero lifetime and is reminiscent of a widely used condition for individual utility functions on health (e.g., Bleichrodt et al., 1997; Miyamoto et al., 1998; Østerdal 2005). It says that if an agent gets zero lifetime, then her health state does not influence the social desirability of the health distribution. Formally,

ZERO: For each $h \in H$ and $i \in N$ such that $t_i = 0$, and $a'_i \in A$, $h \sim [h_{N \setminus \{i\}}, (a'_i, 0)]$.

3 The axiomatic characterizations of two focal PHEFs: QALYs and HYE

The most widely employed way of combining the quality of life and quantity of life derived from a particular health care intervention is by means of QALYs. The following PHEF, which we call *linear QALY*, evaluates population health distributions by means of the unweighted aggregation of individual QALYs in society, or, in other words, by the weighted (through health levels) aggregate time span the distribution yields. Formally,

$$P^q[h_1, \dots, h_n] = P^q[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n q(a_i) t_i, \quad (1)$$

where $q : A \rightarrow [0, 1]$ is an arbitrary function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$.

Our first result says that P^q is characterized by the combination of the basic structural axioms described above plus the following specific axiom, *time invariance at common health*, which refers to the equal value of life gains for persons with common health states.⁷ Formally,

TICH: For each $h \in H$, $c > 0$, and $i, j \in N$, such that $a_i = a_j$,

$$[(a_i, t_i + c), (a_j, t_j), h_{N \setminus \{i, j\}}] \sim [(a_i, t_i), (a_j, t_j + c), h_{N \setminus \{i, j\}}].$$

Our characterization result comes next.

Theorem 1 *The following statements are equivalent:*

1. \succsim is represented by a PHEF satisfying (1).
2. \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TICH.

An alternative way of combining the quality of life and quantity of life derived from a particular health care intervention is by means of *healthy years equivalents* (HYES). The next PHEF, which we call *linear HYE*, evaluates population health distributions by means of the aggregation of individuals' HYES. Formally,

$$P^h[h_1, \dots, h_n] = P^h[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n f(a_i, t_i), \quad (2)$$

where $f : A \times T \rightarrow T$ is a function indicating the HYES for each possible individual, i.e., for each $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}].$$

Our second result characterizes the linear HYE PHEF by adding to the basic structural axioms a weakening of TICH. More precisely, *time invariance at perfect health* is the weakening of the above axiom to the case in which the common health state is precisely the perfect health state. Formally,

TIPH: For each $h \in H$, $c > 0$, and $i, j \in N$, such that $a_i = a_j = a_*$,

$$[(a_i, t_i + c), (a_j, t_j), h_{N \setminus \{i, j\}}] \sim [(a_i, t_i), (a_j, t_j + c), h_{N \setminus \{i, j\}}].$$

The characterization result of the HYE PHEF comes next.

⁷Thus, the axiom conveys an absence of lifetime discrimination: an individual is not less worthy of treatment on the sole grounds that she has a longer lifetime. As a matter of fact, the axiom is very similar, although not identical, to the so-called non-age dependence axiom in Østerdal (2005).

Theorem 2 *The following statements are equivalent:*

1. \succsim is represented by a PHEF satisfying (2).
2. \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TIPH.

As we can observe from the statements of the two theorems, weakening TICH, so that the axiom is restricted to the case in which the common health state is the perfect health state, allows us to move from the (linear) QALY PHEF to the much broader family of (linear) HYE PHEFs.

4 More general PHEFs

Both (linear) PHEFs highlighted in the previous section amount unweighted aggregation across individuals, an aspect usually criticized in the health economics literature by its lack of concern for distributive justice (e.g., Loomes and McKenzie, 1989; Wagstaff, 1991; Dolan, 1998). We present (and characterize) in this section more general PHEFs, which aim to generalize the above two, in order to capture such a concern.

4.1 Power QALYs and HYE

We start with two PHEFs that simply introduce suitable powers in the QALYs (or HYE) each agent in the population gets. The rationale behind this idea can be traced back to Bergson (e.g., Burk, 1936). In a health economics context, power functions of QALYs were introduced, at an individual level, by Pliskin et al., (1980). The so-called Bergsonian approach has also been discussed in the health economics literature by Wagstaff (1991) and Williams (1997), among others.

Formally, the *power QALY* PHEF is defined as:

$$P^{pq}[h_1, \dots, h_n] = P^{pq}[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n q(a_i) t_i^\gamma, \quad (3)$$

where $q : A \rightarrow [0, 1]$ is an arbitrary function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$, and $\gamma \in \mathbb{R}_{++}$ is a positive scalar.

Likewise, the *power HYE* PHEF is defined as:

$$P^{ph}[h_1, \dots, h_n] = P^{ph}[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n f(a_i, t_i)^\gamma, \quad (4)$$

where $\gamma \in \mathbb{R}_{++}$ is a positive scalar, and $f : A \times T \rightarrow T$ is a function indicating the HYE for each possible individual, i.e., for each $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}].$$

As we show next, the above two PHEFs are characterized by combining the following pair of axioms with the basic structural axioms we introduced in Section 2.

The axiom of *time scale independence* says that evaluations should not depend on the variable we use to measure quantity of life (e.g., days, months, years). More precisely, it says that the ranking of a pair of population health distributions does not reverse when all lifetimes are multiplied by a common positive constant.⁸ The individual level counterpart to this axiom is often referred to in the literature as *the constant proportional trade-off* assumption (e.g., Pliskin et al., 1980). Previous experimental studies have provided mixed support for such assumption (e.g., Attema and Brouwer, 2010), with mostly person and context-specific violations. Thus, we believe it may be more acceptable for a planner to assume scale independence in time for social preferences rather than for individual preferences.

TSI: For each $c > 0$, and $h = [(a_i, t_i)_{i \in N}]$, $h' = [(a'_i, t'_i)_{i \in N}]$,

$$h \succsim h' \Rightarrow [(a_i, ct_i)_{i \in N}] \succsim [(a'_i, ct'_i)_{i \in N}].$$

A plausible weakening of the previous axiom, *time scale independence at perfect health*, says that the notion only applies when restricted to perfect health. Formally,

TSIPH: For each $c > 0$, and $h = [(a_*, t_i)_{i \in N}]$, $h' = [(a_*, t'_i)_{i \in N}]$,

$$h \succsim h' \Rightarrow [(a_*, ct_i)_{i \in N}] \succsim [(a_*, ct'_i)_{i \in N}].$$

The characterizations of the power versions of the QALY and HYE models come next.⁹

⁸Østerdal (2005) considers a counterpart of this axiom in his context.

⁹One might find natural to restrict both families to those PHEFs corresponding to $\gamma \leq 1$. It turns out that the following characterization results stated in Theorems 3 and 4 could be enriched to characterize the subsequent families, upon adding a *Pigou-Dalton transfer* axiom stating that a health profile in which two agents at perfect health have different time spans is dominated by the subsequent profile in which those agents keep the same perfect health status, but share a time span equal to the average of the former two.

Theorem 3 *The following statements are equivalent:*

1. \succsim is represented by a PHEF satisfying (3).
2. \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TSI.

Theorem 4 *The following statements are equivalent:*

1. \succsim is represented by a PHEF satisfying (4).
2. \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TSIPH.

It is worth noting that the TSI axiom assumes scale independence for arbitrary health states while the TICH axiom assumes time invariance for persons with common health states only. Other than that, the parallelism between Theorems 2 and 4 mimics the parallelism between Theorems 1 and 3.

4.2 Other generalizations

We conclude with our results by considering two generalized (semi-multiplicative) PHEFs. The first one generalizes the time component in the linear QALY model so that it enters into the PHEF in an arbitrary (not necessarily linear) way. In that sense, it becomes a sort of hybrid between a linear QALY model and the more general structure of the HYE model.¹⁰ Formally,

$$P^{qh}[h_1, \dots, h_n] = P^{qh}[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n q(a_i)w(t_i), \quad (5)$$

where $q : A \rightarrow [0, 1]$ is an arbitrary function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$, and $w : T \rightarrow [0, 1]$ is a strictly increasing function satisfying $w(0) = 0$.

It turns out that this family is singled out when a new axiom, to be defined below, is added to our basic structural axioms. First, we need some prior definitions. A health state $a \in A$ is *non-degenerate* if there exist distributions of life years $(t_i)_{i \in N}$ and $(t'_i)_{i \in N}$ such that $[(a, t_i)_{i \in N}] \approx [(a, t'_i)_{i \in N}]$. For each $h = [(a_i, t_i)_{i \in N}] \in H$ we say that t is the *time egalitarian equivalent*

¹⁰A similar type of generalization was axiomatized by Miyamoto et al., (1998) for a model of individual decision making in health.

if $h \sim [(a_i, t)_{i \in N}]$.¹¹ The axiom, *time egalitarian equivalent invariance for common health* states that time egalitarian equivalents are independent of non-degenerate health states when all agents share them.

TEICH: For each $h = [(a, t_i)_{i \in N}]$, $h' = [(a', t_i)_{i \in N}] \in H$ such that $a, a' \in A$ are non-degenerate, their time egalitarian equivalents coincide.

We can now state the formal result.

Theorem 5 *The following statements are equivalent:*

1. \succsim is represented by a PHEF satisfying (5).
2. \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TE-ICH.

The last PHEF we consider is another natural generalization of the linear QALY PHEF, in which individual QALYs are submitted to an arbitrary (but increasing) function before being aggregated. Formally,

$$P^{gq}[h_1, \dots, h_n] = P^{gq}[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n g(q(a_i)t_i), \quad (6)$$

where $q : A \rightarrow [0, 1]$ is an arbitrary function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$, and $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a strictly increasing real function.

It turns out that this family is singled out when a new axiom called *time scale independence for common health duplets*, is added to our basic structural axioms described above. The axiom states that if we have two health profiles with common duplets then the preference between them is independent of a scaling of the life year component. Formally,

TSICD: For each $c > 0$, and $h = [(a, t)_{i \in N}]$, $h' = [(a', t')_{i \in N}] \in H$,

$$h \succsim h' \Rightarrow [(a, ct)_{i \in N}] \succsim [(a', ct')_{i \in N}].$$

We now state the formal result.

Theorem 6 *The following statements are equivalent:*

1. \succsim is represented by a PHEF satisfying (6).
2. \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TSICD.

¹¹The idea of egalitarian equivalence is deeply rooted in welfare economics. The reader is referred to the excellent monograph by Fleurbaey and Maniquet (2011) for a recent account of the many applications that have been made of this idea in related contexts.

5 Discussion

We have presented in this paper a new axiomatic approach to the evaluation of population health. Contrary to most of the previous axiomatic work within the field, we have considered a model in which no considerations about individual preferences over health have been made. In doing so, we bypass a recurrent criticism regarding the difficulty of assessing individual preferences over health profiles (e.g., Dolan, 2000).¹² We have shown that a framework in which information on individual preferences over health is not available (either for practical or ethical reasons) still allows to make sound decisions over the evaluation of population health.¹³ Somewhat related, even though we assume (as customary) that individual health is determined by two dimensions (quality and quantity) we do not presume that individuals evaluate them in a specific given way. In other words, as opposed to earlier axiomatic contributions on the evaluation of health profiles, we do not assume that the health of an individual is summarized by a number (to be interpreted as the number of QALYs, life years, or “health utility” experienced by a person), but by a duplet referring to the two dimensions.

We have characterized two focal representations of social preferences over population health distributions; namely, the linear QALY and HYE representations. They translate into our context two of the most well-known and employed techniques to measure the benefits of health interventions in cost-utility analyses. Those techniques were initially considered as polars, although some of their differences and similar aspects were addressed (e.g., Mehrez and Gafni, 1989; Culyer and Wagstaff, 1993; Gafni et al., 1993; Bleichrodt, 1995). We have seen in this paper that they share a solid common ground. To wit, their characterizations share many axioms (anonymity, separability, continuity, perfect health superiority, time monotonicity at perfect health, non-desirable death, and the social zero condition) and differ only in one (the linear QALY requires time invariance at common health, whereas the linear HYE only requires time invariance at perfect health).

It is worth mentioning that our aim has been to derive specific representations of social preferences over population health distributions as a result

¹²Furthermore, it has been recently argued in the so-called Global Burden of Diseases, Injuries, and Risk Factors Study that person trade-off (rather than individual preference) information should provide the basis for making priorities.

¹³An alternative, albeit related, framework has been recently proposed by Fleurbaey and Shokkaert (2009), who include a concern for respecting individual preferences by means of a Pareto principle. Such Pareto principle implies that society is indifferent between a status quo and an hypothetical situation in which “healthy equivalent consumptions” are considered. This is reminiscent to our treatment of HYE.

of combining several ethical and operational axioms. We believe our axioms are compelling from a normative viewpoint (although, obviously, some to a higher extent than others). The positive appeal of our axioms has not been tested in this paper. Nevertheless, there exists a wide variety of experimental contributions testing empirically some of the principles (or related ideas) over which our axioms rely in related contexts (e.g., Spencer, 2003; Doctor et al., 2004; Spencer and Robinson, 2007; Turpcu et al., 2011). The test of the precise axioms we use in our specific context is left for further research.¹⁴

As mentioned above, our axiomatic characterizations permit to scrutinize the virtues and shortcomings of the PHEFs we single out by means of comparing the principles (axioms) that drive each of them. In particular, we have seen that the linear QALY model relies on the principle of time invariance at common health, whose normative appeal is mostly restricted to a framework in which the populations being evaluated reflect a cohort of agents with the same age. This indicates that such a framework should be the appropriate one to perform standard QALY-based evaluations of population health, which connects to an old concern in health economics (e.g., Weinstein, 1988). A similar caveat could be made for the linear HYE model, which relies on the weaker (albeit intimately related) axiom of time invariance at perfect health.

We believe that the main relevance of our contribution lies in the lack of mathematical structure we consider for the set of available health states. This opens up future possible extensions of our work linking it to other popular lines of research nowadays, such as the measurement of multidimensional income inequality, poverty or welfare, or the evaluation of population health in a context in which health states worse than dead are allowed.

To conclude, it is also worth commenting on another important aspect of our model. We have not dealt with uncertainty in our analysis.¹⁵ More precisely, we have considered a formulation of the population health evaluation problem which contains no explicit element of risk, and in which we obtain characterizations of population health evaluation functions without assumptions on the social planner’s (or individuals’) risk attitudes. Rather than a limitation, we see this aspect of our analysis as an advantage, as it al-

¹⁴We, nevertheless, would remain cautious about the eventual results of such a venture. As Amos Tversky, one of the leading scholars in the positive approach to decision theory put it, “give me an axiom and I will design a questionnaire so that the majority of the people will reject it” (e.g., Gilboa, 2009).

¹⁵Broome (1993) argues that uncertainty is a complication rather than an essential part of the problem of valuing lives, and it ought not to be introduced into the analysis.

lows us to escape from the usual critiques of the expected utility theory that are normally considered in the health economics literature (e.g., Bleichrodt et al., 2001).

6 Appendix. Proofs

Before proving our theorems, we start with two preliminary results. The first one says that our basic structural axioms imply the following property of *healthy years equivalence*, which says that any population health distribution has a socially equivalent one in which the health outcome of one (and only one) agent is replaced by that of full health, for some quantity of time.¹⁶ Formally,

HYE: For each $h \in H$ and $i \in N$ there exists t_i^* such that $h \sim [h_{N \setminus \{i\}}, (a_*, t_i^*)]$.

Note that the HYE axiom only postulates that, for each agent and health profile, a “healthy years equivalent” exists, but not how it should be determined as a function of the health profile.

Lemma 1 *If \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, and ZERO, then it also satisfies HYE.*

Proof. Let $i \in N$ and $h = [h_{N \setminus \{i\}}, (a_i, t_i)] \in H$. Our aim is to show that there exists $t_i^* \geq 0$ such that

$$h \sim [h_{N \setminus \{i\}}, (a_*, t_i^*)].$$

If $t_i = 0$, it follows, by *ZERO*, that $t_i^* = 0$. Otherwise, suppose that $t_i > 0$, and, by contradiction, that such t_i^* does not exist. Then, $T = A \cup B$, where,

$$A = \{s \in T \mid h \succ [h_{N \setminus \{i\}}, (a_*, s)]\},$$

and

$$B = \{s \in T \mid [h_{N \setminus \{i\}}, (a_*, s)] \succ h\}.$$

By NDD, $0 \in A$ and, thus, $A \neq \emptyset$.

Similarly, by *PHS* and *TMPh*, $t_i + \varepsilon \in B$, for some $\varepsilon > 0$, and thus $B \neq \emptyset$.

By *CONT*, A and B are open sets relative to T .

¹⁶This notion can be traced back to Mehrez and Gafni (1989) who propose it as a plausible way to reflect patient’s preferences over health.

As $A \cap B = \emptyset$, it would follow that T is not a connected set, a contradiction.

Finally, t_i^* is well defined and uniquely determined by SEP and TMPH.

■

Our second preliminary result provides the characterization derived from combining our basic structural axioms. Formally, *separable* PHEFs are defined as follows:

$$P^s[h_1, \dots, h_n] = P^s[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n g(f(a_i, t_i)), \quad (7)$$

where $f : A \times T \rightarrow \mathbb{R}_+$ and $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ are such that:

- f is continuous with respect to its second variable,
- $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$,
- $h \sim [(a_*, f(a_i, t_i))_{i \in N}]$, for each $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$, and
- g is a strictly increasing and continuous function.

Lemma 2 *The following statements are equivalent:*

1. \succsim is represented by a PHEF satisfying (7).
2. \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, and ZERO.

Proof. We focus on the non-trivial implication. Formally, assume \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, and ZERO. By Lemma 1, \succsim satisfies HYE too. Let $h \in H$. By HYE, for each $i \in N$, there exists t_i^* such that $h \sim [h_{N \setminus \{i\}}, (a_*, t_i^*)]$. By SEP, t_i^* only depends on (a_i, t_i) (and, thus, is independent of the remaining duplets of the profile). Thus, for each $i = 1, \dots, n$, let $f_i : A \times T \rightarrow \mathbb{R}$ be defined such that $f_i(a_i, t_i) = t_i^*$, for each $(a_i, t_i) \in A \times T$. By ANON, $f_i(\cdot, \cdot) \equiv f_j(\cdot, \cdot) \equiv f(\cdot, \cdot)$, for each $i, j \in N$. By TMPH and PHS, $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$ and, by CONT, f is a continuous function with respect to its second variable. Furthermore,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}],$$

which implicitly says that social preferences only depend on the profile of healthy years equivalents, and, by CONT, they do so continuously. It also follows that the range of f is a connected subset of \mathbb{R} . By Theorem 3 in

Debreu (1960), there exists a strictly increasing and continuous function $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ such that

$$h \succsim h' \iff \sum_{i=1}^n g(f(a_i, t_i)) \geq \sum_{i=1}^n g(f(a'_i, t'_i)),$$

which concludes the proof. ■

6.1 Proof of Theorem 1

We focus on the non-trivial implication, i.e., $2 \rightarrow 1$. Formally, assume \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TICH. Then, by Lemma 2, there exists $f : A \times T \rightarrow \mathbb{R}_+$ and $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ are such that:

- f is continuous with respect to its second variable,
- $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, and
- g is a strictly increasing and continuous function.

Furthermore, for each $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}],$$

and, for each $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$, and $h' = [(a'_1, t'_1), \dots, (a'_n, t'_n)] \in H$,

$$h \succsim h' \iff \sum_{i=1}^n g(f(a_i, t_i)) \geq \sum_{i=1}^n g(f(a'_i, t'_i)).$$

Without loss of generality, let us assume that $g(f(\hat{a}, 0)) = 0$ for some $\hat{a} \in A$.

Let $\bar{a} \in A$ be an arbitrary health state. Then, by iterated application of TICH, and the transitivity of \succsim ,

$$\sum_{i=1}^n g(f(\bar{a}, t_i)) = g\left(f\left(\bar{a}, \sum_{i=1}^n t_i\right)\right) + (n-1)g(f(\bar{a}, 0)).$$

By ZERO, $g(f(\bar{a}, 0)) = g(f(\hat{a}, 0)) = 0$. Thus,

$$\sum_{i=1}^n g(f(\bar{a}, t_i)) = g\left(f\left(\bar{a}, \sum_{i=1}^n t_i\right)\right).$$

It then follows that $g(f(\bar{a}, \cdot))$ satisfies

$$g(f(\bar{a}, t_1 + t_2)) = g(f(\bar{a}, t_1)) + g(f(\bar{a}, t_2)) \quad (8)$$

for any $t_1, t_2 \in T$, which is precisely one of Cauchy's canonical functional equations. As $g(f(\bar{a}, \cdot))$ is a continuous function, it follows that the unique solutions to such equation are the linear functions (e.g., Aczel, 2006; page 34). More precisely, there exists a function $\hat{q} : A \rightarrow \mathbb{R}$ such that

$$g(f(\bar{a}, t)) = \hat{q}(\bar{a})t,$$

for each $\bar{a} \in A$, and $t \in T$. By PHS and TMPH, it follows that $0 \leq \hat{q}(a_*)$ and $\hat{q}(a_i) \leq \hat{q}(a_*)$, for each $a_i \in A$. We now show that $\hat{q}(a_i) \geq 0$, for each $a_i \in A$. To do so, let $a_i \in A$. By Lemma 1, there exists $t_i \in T$ such that $\hat{q}(a_i) = g(f(a_i, 1)) = g(f(a_*, t_i))$. By TMPH and ZERO, $g(f(a_*, t_i)) \geq g(f(a_*, 0)) = g(f(\hat{a}, 0)) = 0$. Altogether, it says that $\hat{q}(a_i) \geq 0$. To conclude, let $q : A \rightarrow \mathbb{R}$ be such that $q(a) = \frac{\hat{q}(a)}{\hat{q}(a_*)}$, for each $a \in A$. Then, it follows from the above that $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$. Thus, \succsim is represented by a PHEF satisfying (1), as desired. ■

6.2 Proof of Theorem 2

We focus on the non-trivial implication, i.e., $2 \rightarrow 1$. Formally, assume \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TIPH. Then, by Lemma 2, there exists $f : A \times T \rightarrow \mathbb{R}_+$ continuous with respect to its second variable, and satisfying that $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, such that, for each $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}].$$

Let $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$, and $h' = [(a'_1, t'_1), \dots, (a'_n, t'_n)] \in H$. Then, by iterated application of TIPH, and the transitivity of \succsim ,

$$h \succsim h' \iff [(a_*, \sum_{i \in N} f(a_i, t_i)), (a_*, 0)_{k \in N \setminus \{i\}}] \succsim [(a_*, \sum_{i \in N} f(a'_i, t'_i)), (a_*, 0)_{k \in N \setminus \{i\}}].$$

By PHS, and the transitivity of \succsim ,

$$h \succsim h' \iff \sum_{i \in N} f(a_i, t_i) \geq \sum_{i \in N} f(a'_i, t'_i),$$

as desired. ■

6.3 Proof of Theorem 3

We focus on the non-trivial implication, i.e., $2 \rightarrow 1$. Formally, assume \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TSI. Then, by

Lemma 2, there exists $f : A \times T \rightarrow \mathbb{R}_+$ continuous with respect to its second variable, and satisfying that $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, such that, for each $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}].$$

Step 1. We claim that for each $(a, t), (a', t') \in A \times T$, and $c > 0$,

$$f(a, t) \geq f(a', t') \iff f(a, ct) \geq f(a', ct').$$

Indeed, let $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$ and $c > 0$. Denote $h^c = [(a_1, ct_1), \dots, (a_n, ct_n)]$ and let $(a, t), (a', t') \in A \times T$. By (7),

$$[(a, t), h_{N \setminus \{i\}}] \succsim [(a', t'), h_{N \setminus \{i\}}] \iff f(a, t) \geq f(a', t'),$$

and

$$[(a, ct), h_{N \setminus \{i\}}^c] \succsim [(a', ct'), h_{N \setminus \{i\}}^c] \iff f(a, ct) \geq f(a', ct').$$

By TSI,

$$[(a, t), h_{N \setminus \{i\}}] \succsim [(a', t'), h_{N \setminus \{i\}}] \iff [(a, ct), h_{N \setminus \{i\}}^c] \succsim [(a', ct'), h_{N \setminus \{i\}}^c].$$

The transitivity of \succsim concludes.

Step 2. We now claim the following. Let $\bar{q} : A \rightarrow \mathbb{R}$ be such that $\bar{q}(a) = f(a, 1)$, for each $a \in A$. Then,

$$f(a, t) \geq f(a', t') \iff \bar{q}(a)t \geq \bar{q}(a')t',$$

for each $(a, t), (a', t') \in A \times T$.

Indeed, by definition, $f(a_i, 1) = f(a_*, \bar{q}(a_i))$. By Step 1,

$$f(a, t) = f(a', t') \iff f(a, ct) = f(a', ct').$$

Thus, $f(a_i, t_i) = \bar{q}(a_i)t_i$, as desired.

Step 3. We claim there exists $\gamma > 0$ such that, for each $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$, and $h' = [(a'_1, t'_1), \dots, (a'_n, t'_n)] \in H$,

$$h \succsim h' \iff \sum_{i=1}^n (\bar{q}(a_i)t_i)^\gamma \geq \sum_{i=1}^n (\bar{q}(a'_i)t'_i)^\gamma.$$

Let P denote the PHEF defined by¹⁷

$$P[h_1, \dots, h_n] = P[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n g(\bar{q}(a_i)t_i).$$

By Step 2, $f(\cdot, \cdot)$ is a monotonic transformation of the function $\tau : A \times T \rightarrow \mathbb{R}$ defined by $\tau(a, t) = \bar{q}(a)t$, for each $(a, t) \in A \times T$. Then, by (7), P represents \succsim . By TSI,

$$\sum_{i=1}^n g(\bar{q}(a_i)t_i) \geq \sum_{i=1}^n g(\bar{q}(a'_i)t'_i) \iff \sum_{i=1}^n g(\bar{q}(a_i)ct_i) \geq \sum_{i=1}^n g(\bar{q}(a'_i)ct'_i),$$

for each $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$, $h' = [(a'_1, t'_1), \dots, (a'_n, t'_n)] \in H$ and $c > 0$.

By Bergson and Samuelson (e.g., Burk, 1936; Samuelson, 1965), there are only three possible functional forms for P :

- $P[h_1, \dots, h_n] = P[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n \alpha_i (\bar{q}(a_i)t_i)^\gamma,$
- $P[h_1, \dots, h_n] = P[(a_1, t_1), \dots, (a_n, t_n)] = -\sum_{i=1}^n \alpha_i (\bar{q}(a_i)t_i)^\delta,$
- $P[h_1, \dots, h_n] = P[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n \alpha_i \log(\bar{q}(a_i)t_i),$

for some $\gamma > 0$, $\delta < 0$ and sequence $\{\alpha_i\}$ such that $\alpha_i > 0$ for each $i \in N$, and for each $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$ such that $t_i > 0$, for each $i \in N$. It is straightforward to show that the last two functional forms cannot be continuously extended to the whole domain H , in which zero time spans are allowed. By ANON, $\alpha_i = \alpha_j$ for each $i, j \in N$. Finally, let $q : A \rightarrow \mathbb{R}$ be such that $q(a) = \bar{q}(a)^\gamma$, for each $a \in A$. Altogether, we have that $P = P^{pq}$, as desired. ■

6.4 Proof of Theorem 4

We focus on the non-trivial implication, i.e., $2 \rightarrow 1$. Formally, assume \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TSI. Then, by Lemma 2, there exists $f : A \times T \rightarrow \mathbb{R}_+$ and $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ are such that:

- f is continuous with respect to its second variable,
- $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, and

¹⁷The proof of this step follows closely the argument of the proof of Theorem 8 in Østerdal (2005).

- g is a strictly increasing and continuous function.

Furthermore, for each $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}],$$

and, for each $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$, and $h' = [(a'_1, t'_1), \dots, (a'_n, t'_n)] \in H$,

$$h \succsim h' \iff \sum_{i=1}^n g(f(a_i, t_i)) \geq \sum_{i=1}^n g(f(a'_i, t'_i)).$$

By TSIPH,

$$\sum_{i=1}^n g(f(a_i, t_i)) \geq \sum_{i=1}^n g(f(a'_i, t'_i)) \iff \sum_{i=1}^n g(f(a_i, ct_i)) \geq \sum_{i=1}^n g(f(a'_i, ct'_i)),$$

for each $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$, $h' = [(a'_1, t'_1), \dots, (a'_n, t'_n)] \in H$ and $c > 0$.

Let P be a PHEF representing \succsim . As in the proof of Theorem 3, by Bergson and Samuelson, ANON, and the requirement that P represent \succsim in the whole domain H , it follows that

$$P[h_1, \dots, h_n] = P[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^n (f(a_i, t_i))^\gamma,$$

for some $\gamma > 0$, and for each $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$, which concludes the proof. ■

6.5 Proof of Theorem 5

We focus on the non-trivial implication, i.e., $2 \rightarrow 1$. Formally, assume \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TEICH. Then, by Lemma 2, there exists $f : A \times T \rightarrow \mathbb{R}_+$ continuous with respect to its second variable, and satisfying that $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, such that, for each $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}].$$

Let $w : T \rightarrow \mathbb{R}_+$ be such that $w(t) = f(a_*, t)$, for each $t \in T$.¹⁸ By ZERO, $w(0) = 0$. Let a be an arbitrary non-degenerate health state and

¹⁸Part of the ensuing proof parallels an argument in the proof of Theorem 3.1 in Miyamoto et al., (1998).

consider the resulting health profile $h \in H$ in which a fraction p of the population enjoys a for a period t_1 whereas the rest enjoy it for a period $t_2 \neq t_1$. Let $t^*(t_1, t_2)$ be the time egalitarian equivalent. By (7), $pf(a, t_1) + (n - p)f(a, t_2) = nf(a, t^*(t_1, t_2))$, and, thus, $\frac{p}{n}f(a, t_1) + (1 - \frac{p}{n})f(a, t_2) = f(a, t^*(t_1, t_2))$. By TEICH we have $\frac{p}{n}w(t_1) + (1 - \frac{p}{n})w(t_2) = w(t^*(t_1, t_2))$. As this holds for any $t_1, t_2 \geq 0$, $t_1 \neq t_2$, the function $f(a, t)$, for fixed a , is an affine transformation of each $w(t)$. Hence, there are functions $A : A \rightarrow \mathbb{R}_+$, $B : A \rightarrow \mathbb{R}_+$, such that

$$f(a, t) = A(a)w(t) + B(a).$$

By ZERO we get $B(a) = 0$ for each $a \in A$. By HYE, $A(a) > 0$ for each non-degenerate state $a \in A$. Then, define the function $q : A \rightarrow \mathbb{R}_+$ by $q(a) = A(a)$ for each non-degenerate state $a \in A$ and $q(a) = 0$ otherwise. As a result, $f(a, t) = q(a)w(t)$. By TMPH, w is strictly increasing. By HYE, $q(a) \geq 0$ for each $a \in A$. Finally, by TMPH, and the fact that $h \sim [(a_*, f(a_i, t_i))_{i \in N}]$, we obtain that $q(a) \leq q(a_*) = 1$, for each $a \in A$, which concludes the proof. ■

6.6 Proof of Theorem 6

We focus on the non-trivial implication, i.e., $2 \rightarrow 1$. Formally, assume \succsim satisfies ANON, SEP, CONT, PHS, TMPH, NDD, ZERO and TSICD. Then, by Lemma 2, there exists $f : A \times T \rightarrow \mathbb{R}_+$ and $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ are such that:

- f is continuous with respect to its second variable,
- $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, and
- g is a strictly increasing and continuous function.

Furthermore, for each $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}],$$

and, for each $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$, and $h' = [(a'_1, t'_1), \dots, (a'_n, t'_n)] \in H$,

$$h \succsim h' \iff \sum_{i=1}^n g(f(a_i, t_i)) \geq \sum_{i=1}^n g(f(a'_i, t'_i)).$$

Step 1. We claim that for each $(a, t), (a', t') \in A \times T$, and $c > 0$,

$$f(a, t) \geq f(a', t') \iff f(a, ct) \geq f(a', ct').$$

Indeed, let $h = [(a, t), \dots, (a, t)]$, $h' = [(a', t'), \dots, (a', t')] \in H$ and $c > 0$. Denote $h_c = [(a, ct), \dots, (a, ct)]$ and $h'_c = [(a', ct'), \dots, (a', ct')]$. By (7),

$$h \succsim h' \iff f(a, t) \geq f(a', t'),$$

and

$$h_c \succsim h'_c \iff f(a, ct) \geq f(a', ct').$$

By TSICD, the claim follows.

Step 2. Let $\bar{q} : A \rightarrow \mathbb{R}$ be such that $\bar{q}(a) = f(a, 1)$, for each $a \in A$. We claim that

$$f(a, t) \geq f(a', t') \iff \bar{q}(a)t \geq \bar{q}(a')t',$$

for each $(a, t), (a', t') \in A \times T$.

The proof is similar to that of Step 2 in the proof of Theorem 3.

By Step 2, $f(\cdot, \cdot)$ is a monotonic transformation of the function $\tau : A \times T \rightarrow \mathbb{R}$ defined by $\tau(a, t) = \bar{q}(a)t$, for each $(a, t) \in A \times T$. Then, by the above, P^{gq} represents \succsim , as desired. ■

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